university of groningen

System Identification using Energy-Bounded Noise Models

A Full Characterization of Chebyshev Centers and Radii

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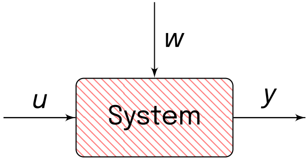
System Identification



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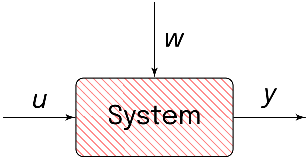

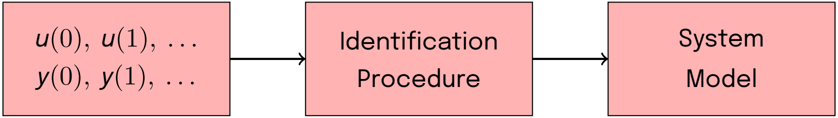

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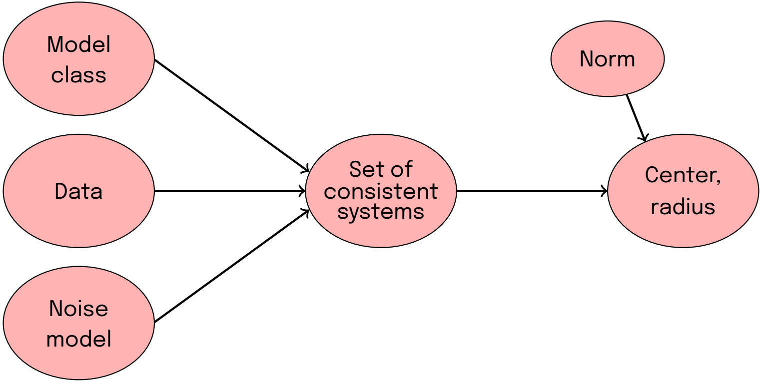
   2.2 Deterministic noise models

      2.2.1 Assumptions: Unknown but bounded.

      2.2.2 Methods: Set of consistent systems, and its center, radius, etc.

System Identification

using Set-Membership Noise Models



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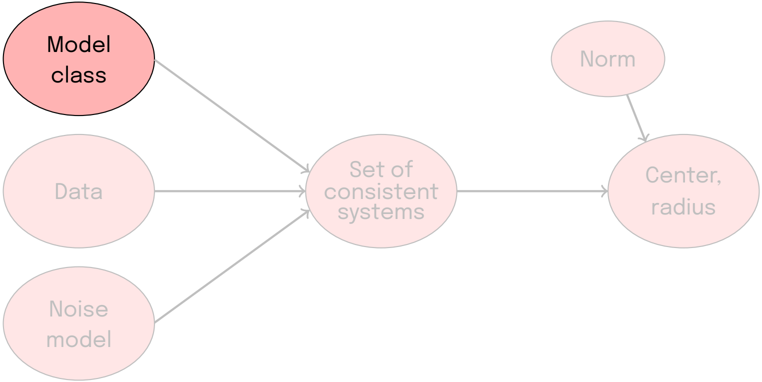

Model Class

We consider auto-regressive models of the following form:

$$y(t+L) = \sum_{i=0}^{L-1} P_i \, y(t+i) + \sum_{j=0}^{M} Q_j \, u(t+j) + w(t)$$

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- We assume that $M \leq L$ (causality).

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$$P_{\text{true}} := \begin{bmatrix} P_0 & \cdots & P_{L-1} \end{bmatrix} \quad \text{and} \quad Q_{\text{true}} := \begin{bmatrix} Q_0 & \cdots & Q_M \end{bmatrix}.$$

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The true system $(P_{\mathrm{true}}, Q_{\mathrm{true}})$ is unknown.

System Identification

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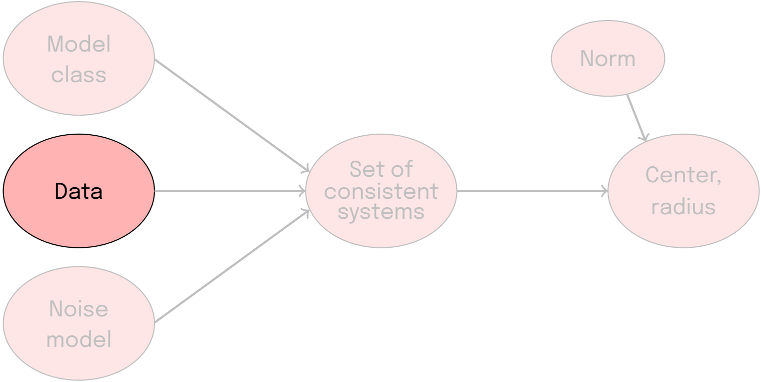

Data

We collect the input-output data

$$u(0), \ldots, u(T+M-L), \quad y(0), \ldots, y(L), \ldots, y(T-1), y(T),$$

where $T \geq L$.

Data

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$$u(0),\dots,u(T+M-L),\quad y(0),\dots,y(L),\dots,y(T-1),y(T),$$

where $T \geq L$. We construct the following data matrices:

$$U_- := \begin{bmatrix} u(0) & u(1) & \cdots & u(T-L) \\ \vdots & \vdots & \ddots & \vdots \\ u(M) & u(M+1) & \cdots & u(T+M-L) \end{bmatrix},$$

$$Y_- := \begin{bmatrix} y(0) & y(1) & \cdots & y(T-L) \\ \vdots & \vdots & \ddots & \vdots \\ y(L-1) & y(L) & \cdots & y(T-1) \end{bmatrix}, \text{ and}$$

$$Y_+ := \begin{bmatrix} y(L) & y(L+1) & \cdots & y(T) \end{bmatrix}.$$

System Identification

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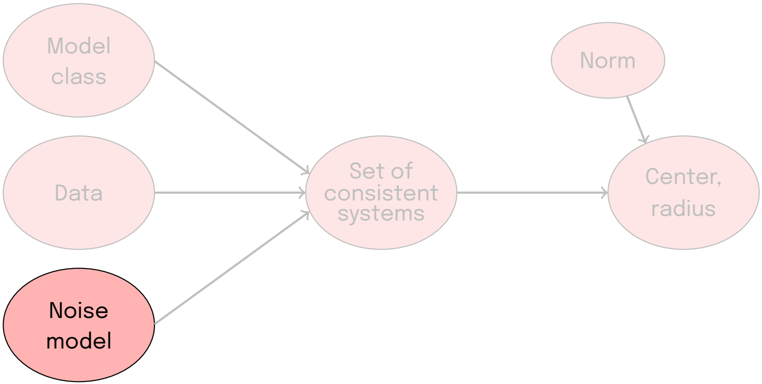

Noise Model

We define $\quad W_- := \begin{bmatrix} w(0) & w(1) & \cdots & w(T-L) \end{bmatrix}.$

Assumption

The noise matrix W_- satisfies the QMI

$$\begin{bmatrix} I \\ W_-^\top \end{bmatrix}^\top \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^\top & \Phi_{22} \end{bmatrix} \begin{bmatrix} I \\ W_-^\top \end{bmatrix} \geq 0$$

for a known symmetric matrix Φ with $\Phi_{22} < 0$ and $\Phi|\Phi_{22} := \Phi_{11} - \Phi_{12}\Phi_{22}^{-1}\Phi_{12}^\top \geq 0$.

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Examples: $\Sigma_{t=0}^{T-L+1} w(t)w(t)^\top = W_- W_-^\top \leq \varepsilon^2 I$

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for a known symmetric matrix Φ with $\Phi_{22} < 0$ and $\Phi|\Phi_{22} := \Phi_{11} - \Phi_{12}\Phi_{22}^{-1}\Phi_{12}^\top \geq 0$.

Examples: $(W_- - W_0)\Phi_{22}(W_- - W_0)^\top \leq \Phi_{11} I$

Noise Model

For a symmetric matrix $\Pi \in \mathbb{R}^{(p+q)\times(p+q)}$, we denote a QMI-induced set by

$$\mathcal{Z}_p(\Pi) := \left\{ Z \in \mathbb{R}^{p\times q} : \begin{bmatrix} I \\ Z \end{bmatrix}^\top \Pi \begin{bmatrix} I \\ Z \end{bmatrix} \geq 0 \right\}.$$

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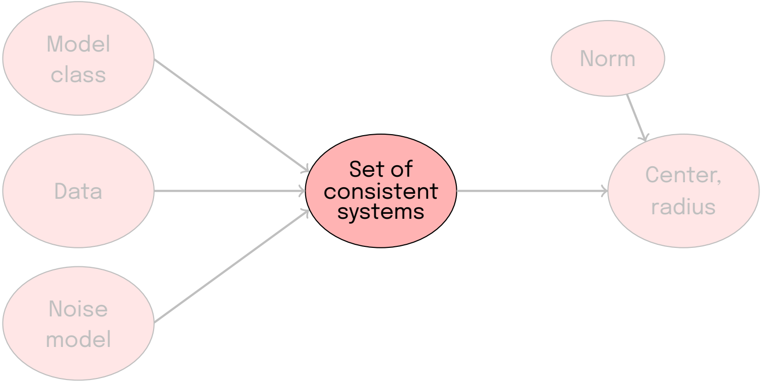
Using the above notation, the noise model is

$$W_-^\top \in \mathcal{Z}_{T-L+1}(\Phi)$$

with $\Phi_{22} < 0$ and $\Phi|\Phi_{22} \geq 0$.

System Identification

using Set-Membership Noise Models



Set of Consistent Systems

A consistent system is a pair of real matrices (P, Q) satisfying

$$Y_+ = PY_- + QU_- + W_-$$

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We define the set of consistent systems as

$$\Sigma := \{(P, Q) : (Y_+ - PY_- - QU_-)^\top \in \mathcal{Z}_{T-L+1}(\Phi)\}.$$

Set of Consistent Systems

Define $s := Lp + (M+1)m$ and

$$N := \begin{bmatrix} I & Y_+ \\ 0 & -Y_- \\ 0 & -U_- \end{bmatrix} \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^\top & \Phi_{22} \end{bmatrix} \begin{bmatrix} I & Y_+ \\ 0 & -Y_- \\ 0 & -U_- \end{bmatrix}^\top .$$

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Proposition

$$(P, Q) \in \Sigma \quad \iff \quad \begin{bmatrix} P & Q \end{bmatrix}^\top \in \mathcal{Z}_s(N)$$

System Identification

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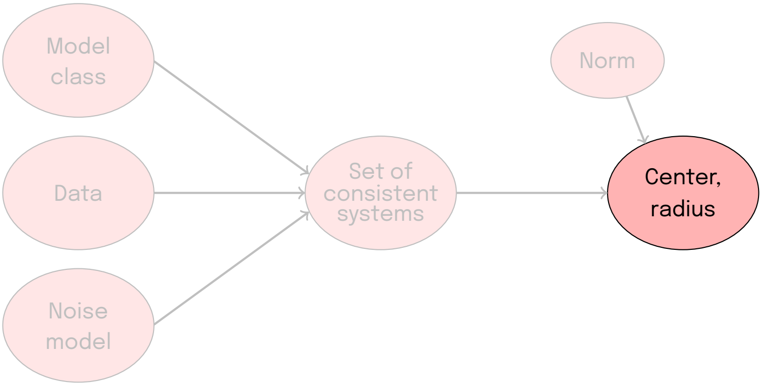
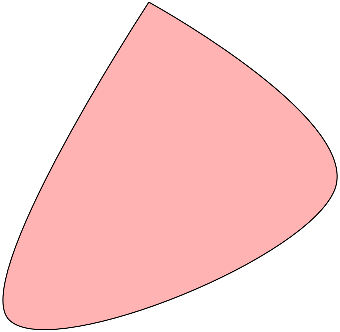


Center, Radius, and Diameter of Sets

For a compact $\mathcal{Z}_s(N)$ and a norm $|\cdot|$,



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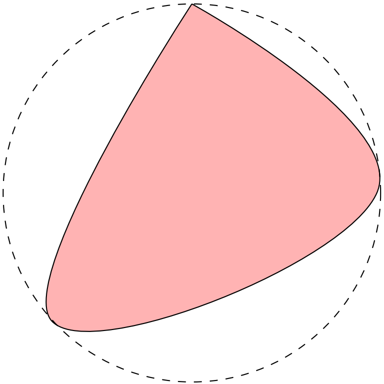
For a compact $\mathcal{Z}_s(N)$ and a norm $|\cdot|$, consider a smallest ball containing $\mathcal{Z}_s(N)$.



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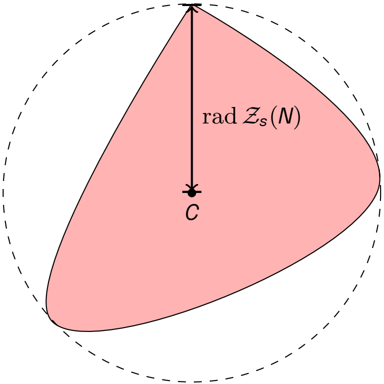

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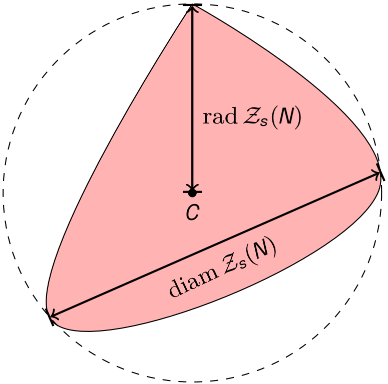

Center, Radius, and Diameter of Sets

For a compact $\mathcal{Z}_s(N) \subset \mathbb{R}^{p \times s}$, the Chebyshev radius is

$$\operatorname{rad} \mathcal{Z}_s(N) := \min_{C \in \mathbb{R}^{p \times s}} \max_{Z \in \mathcal{Z}_s(N)} |C - Z|,$$

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the set of Chebyshev centers is

$$\operatorname{cent} \mathcal{Z}_s(N) := \{C \in \mathbb{R}^{p\times s} : |C - Z| \leq \operatorname{rad} \mathcal{Z}_s(N),\ \forall Z \in \mathcal{Z}_s(N)\},$$

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and the diameter is

$$\operatorname{diam} \mathcal{Z}_s(N) := \max_{Z_1,Z_2\in\mathcal{Z}_s(N)} |Z_1 - Z_2|.$$

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Problem Statement

Given a norm $\|\cdot\|$ and the input-output data with $\begin{bmatrix} Y_- \\ U_- \end{bmatrix}$ of full row rank

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Given a norm $|\cdot|$ and the input-output data with $\begin{bmatrix} Y_- \\ U_- \end{bmatrix}$ of full row rank, find

1. a Chebyshev center,
2. the Chebyshev radius,
3. and the diameter of $\mathcal{Z}_s(N)$.

System Identification

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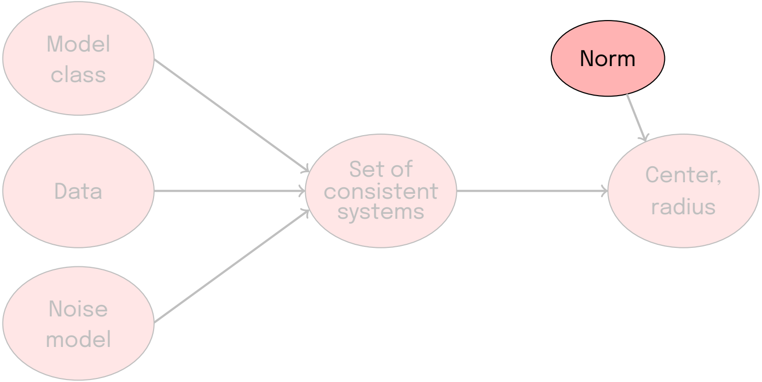

Unitarily Invariant Norms

Definition

A matrix norm $\| \cdot \|$ on $\mathbb{C}^{n\times m}$ is called unitarily invariant if

$$\|UMV\| = \|M\|$$

for all $M \in \mathbb{C}^{n\times m}$ and all unitary $U \in \mathbb{C}^{n\times n}$ and $V \in \mathbb{C}^{m\times m}$.

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Examples : Spectral norm, Frobenius norm, Schatten p-norms, Ky Fan k-norms.

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1. g is a norm,
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3. $g(x_1, \ldots, x_n) = g(|x_1|, \ldots, |x_n|)$.

Unitarily Invariant Norms

For $M \in \mathbb{C}^{n \times m}$, let $\sigma_1(M) \geq \sigma_2(M) \geq \cdots \geq \sigma_{\min\{n,m\}}(M)$ denote its singular values.
We define $\sigma(M) := [\sigma_1(M), \ldots, \sigma_{\min\{n,m\}}(M)]^\top$.
Also, we define $\sigma_{[p]}(M) := [\sigma_1(M), \ldots, \sigma_p(M)]^\top$, where $p \leq \min\{n, m\}$.

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von Neumann's Theorem (1937)

Unitarily invariant norms are exactly symmetric gauge functions of the vector of singular values.

Main Results

Recall that:

$$N = \begin{bmatrix} I & Y_+ \\ 0 & -Y_- \\ 0 & -U_- \end{bmatrix} \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{12}^\top & \Phi_{22} \end{bmatrix} \begin{bmatrix} I & Y_+ \\ 0 & -Y_- \\ 0 & -U_- \end{bmatrix}^\top$$

Main Results

We partition N as a 2×2 block-form matrix:

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$$= \left[\begin{array}{c|c} \begin{bmatrix} I \\ Y_+^\top \end{bmatrix}^\top \Phi \begin{bmatrix} I \\ Y_+^\top \end{bmatrix} & -\begin{bmatrix} I \\ Y_+^\top \end{bmatrix}^\top \begin{bmatrix} \Phi_{12} \\ \Phi_{22} \end{bmatrix} \begin{bmatrix} Y_- \\ U_- \end{bmatrix}^\top \\ \hline -\begin{bmatrix} Y_- \\ U_- \end{bmatrix}^\top \begin{bmatrix} \Phi_{12} \\ \Phi_{22} \end{bmatrix}^\top \begin{bmatrix} I \\ Y_+^\top \end{bmatrix} & \begin{bmatrix} Y_- \\ U_- \end{bmatrix} \Phi_{22} \begin{bmatrix} Y_- \\ U_- \end{bmatrix}^\top \end{array}\right]$$

Main Results

Now, let's define partitions of N as follows:

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Theorem

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2. $\operatorname{diam} \mathcal{Z}_s(N) = 2 \operatorname{rad} \mathcal{Z}_s(N)$.
3. $-N_{22}^{-1}N_{21} \in \operatorname{cent} \mathcal{Z}_s(N)$.

Remarks

- **The common center:** A Chebyshev center with respect to unitarily invariant norms:

$$\begin{bmatrix}\hat{P} & \hat{Q}\end{bmatrix} := -N_{12}N_{22}^{-1} = \begin{bmatrix} I \\ Y_+^\top \end{bmatrix}^\top \begin{bmatrix} \Phi_{12} \\ \Phi_{22} \end{bmatrix} \begin{bmatrix} Y_- \\ U_- \end{bmatrix}^\top \left(\begin{bmatrix} Y_- \\ U_- \end{bmatrix} \Phi_{22} \begin{bmatrix} Y_- \\ U_- \end{bmatrix}^\top \right)^{-1}.$$

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- **The common center:** A Chebyshev center with respect to unitarily invariant norms:

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- **Least-squares:** The presented Chebyshev center is also a solution to

$$\operatorname*{argmin}_{P,Q} |(Y_+ - PY_- - QU_- - \Phi_{12}\Phi_{22}^{-1})(-\Phi_{22})^{\frac{1}{2}}|_{\mathrm{F}}.$$

Conclusions and Futher Works

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- Future works: Volume of the set; experiment design.