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# Data-Driven Stabilization Using Prior Knowledge on Stabilizability and Controllability

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# Introduction



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\*C. De Persis, P. Tesi, “Formulas for data-driven control: Stabilization, optimality, and robustness,” *IEEE Transactions on Automatic Control*, vol. 65, no. 3, pp. 909–924, 2020.



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\*H. J. van Waarde, J. Eising, H. L. Trentelman, M. K. Camlibel, “Data informativity: A new perspective on data-driven analysis and control,” *IEEE Transactions on Automatic Control*, vol. 65, no. 11, pp. 4753–4768, 2020.



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  - ▶ **Known bounds** on the system parameters (Berberich et al., 2020)\*.

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\*J. Berberich, C. W. Scherer, F. Allgöwer, “Combining prior knowledge and data for robust controller design,” *IEEE Transactions on Automatic Control*, vol. 68, no. 8, pp. 4618–4633, 2022.



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\*H. Huang, M. K. Camlibel, R. Carloni, H. J. van Waarde, “Data-driven stabilization of polynomial systems using density functions,” *arXiv preprint*, 2025.



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- ▷ Prior knowledge in **system identification**:
  - ▶ Stability (van Gestel et al., 2002, Lacy & Bernstein, 2003)
  - ▶ Eigenvalue constraints (Miller & De Callafon, 2013)
  - ▶ Positivity (De Santis & Farina, 2002)
  - ▶ Passivity (Goethals et al., 2003, Shali & van Waarde, 2024)



# Problem Formulation

Consider the **true system**

$$x(t+1) = A_{\text{true}}x(t) + B_{\text{true}}u(t), \quad (\star)$$

where  $x(t) \in \mathbb{R}^n$  and  $u(t) \in \mathbb{R}^m$ .



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- ▷  $(A_{\text{true}}, B_{\text{true}}) \in \Sigma := \mathbb{R}^{n \times n} \times \mathbb{R}^{n \times m}$  is unknown.
- ▷ We have access to **input-state data** of the form

$$\mathcal{D} := ([u(0) \quad u(1) \quad \cdots \quad u(T-1)], [x(0) \quad x(1) \quad \cdots \quad x(T)]).$$



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- ▷ We denote the **data matrices**

$$X_- := [x(0) \quad x(1) \quad \cdots \quad x(T-1)], \quad U_- := [u(0) \quad u(1) \quad \cdots \quad u(T-1)], \\ X_+ := [x(1) \quad x(2) \quad \cdots \quad x(T)].$$



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## *Definition*

We say  $\mathcal{D}$  is informative for stabilization if there exists a  $K \in \mathbb{R}^{m \times n}$  such that  $A + BK$  is Schur for all  $(A, B) \in \Sigma_{\mathcal{D}}$ .



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## *Proposition* (van Waarde et al., 2020)

The data  $\mathcal{D}$  is informative for stabilization **if and only if** there exists  $\Theta \in \mathbb{R}^{T \times n}$  such that

$$X_- \Theta = \Theta^\top X_-^\top \quad \text{and} \quad \begin{bmatrix} X_- \Theta & X_+ \Theta \\ \Theta^\top X_-^\top & X_- \Theta \end{bmatrix} > 0. \quad (*)$$



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Moreover,  $K = U_- \Theta (X_- \Theta)^{-1}$  is such that  $A + BK$  is Schur for all  $(A, B) \in \Sigma_{\mathcal{D}}$ .



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The data  $\mathcal{D}$  is called  **$\Sigma_{\text{pk}}$ -informative for stabilization** if there exists a  $K \in \mathbb{R}^{m \times n}$  such that  $A + BK$  is Schur for all  $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\text{pk}}$ .



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- ▷ We focus on two types of prior knowledge  $\Sigma_{\text{pk}} = \Sigma_{\text{cont}}$  and  $\Sigma_{\text{pk}} = \Sigma_{\text{stab}}$ .



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## *Problem Statement*

Find necessary and sufficient conditions under which  $\mathcal{D}$  is

- $\Sigma_{\text{cont}}$ -informative for stabilization;
- $\Sigma_{\text{stab}}$ -informative for stabilization.

# Controllability as Prior Knowledge



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- (a)  $\mathcal{D}$  is  $\Sigma_{\text{cont}}$ -informative for stabilization.
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Moreover, if  $K$  is such that  $A + BK$  is Schur for all  $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\text{cont}}$ , then  $A + BK$  is Schur for all  $(A, B) \in \Sigma_{\mathcal{D}}$ .



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▷ Prior knowledge on controllability is **not** helpful!



# Stabilizability as Prior Knowledge

*Theorem* (Full-rank state data)

Suppose that  $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{stab}}$  and  $\text{rank } X_- = n$ .



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▷ If  $(A_{\text{true}}, B_{\text{true}})$  is not controllable, and  $x(0) = 0$ , then

$$\text{rank } X_- \neq n,$$

no matter what the input signal is.



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- (a)  $\text{im } X_+ \subseteq \text{im } X_-$ ,
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▷ Let  $r := \text{rank } X_-$ ,  $S \in \mathbb{R}^{n \times n}$  be nonsingular, and  $\hat{X}_- \in \mathbb{R}^{r \times T}$  be of full row rank such that  $SX_- = \begin{bmatrix} \hat{X}_- \\ 0 \end{bmatrix}$ .



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- ▷ Let  $\hat{X}_+ \in \mathbb{R}^{r \times n}$  be defined as  $\hat{X}_+ := [I_r \quad 0] SX_+$ .



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*Proposition* (Data-driven feedback design with rank-deficient state data)

Suppose that  $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{stab}}$ ,  $\mathcal{D}$  is  $\Sigma_{\text{stab}}$ -informative for stabilization, and  $\text{rank } X_- < n$ . Then, the following statements hold:

(a) There exists  $\Theta \in \mathbb{R}^{T \times r}$  such that the following LMI is feasible:

$$\hat{X}_- \Theta = \Theta^\top \hat{X}_-^\top \quad \text{and} \quad \begin{bmatrix} \hat{X}_- \Theta & \hat{X}_+ \Theta \\ \Theta^\top \hat{X}_+^\top & \hat{X}_- \Theta \end{bmatrix} > 0. \quad (\star)$$



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(b) Suppose that  $\Theta$  satisfies LMI  $(\star)$ . Let  $K = [K_1 \quad K_2] S$ , where  $K_1 = U_- \Theta (\hat{X}_- \Theta)^{-1}$  and  $K_2 \in \mathbb{R}^{m \times (n-r)}$  is arbitrary. Then,  $A + BK$  is Schur for all  $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\text{stab}}$ .



# Example

Consider the following system:

$$A_{\text{true}} = \begin{bmatrix} 0.9429 & 0.0473 & 0.0012 \\ 0.0473 & 0.9524 & 0.0476 \\ 0 & 0 & 0.9512 \end{bmatrix},$$
$$B_{\text{true}} = \begin{bmatrix} 0.0024 \\ 0.0976 \\ 0 \end{bmatrix}.$$

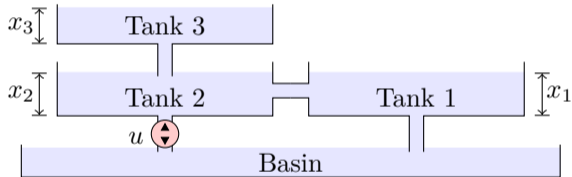


Figure: Three-tank system.



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- ▷ Monte Carlo simulations with 1000 scenarios;

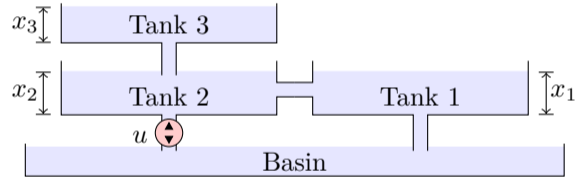


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- ▷ Monte Carlo simulations with 1000 scenarios;
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  - ▶ We simulate the system from  $t = 0$  to  $t = 100$ ;

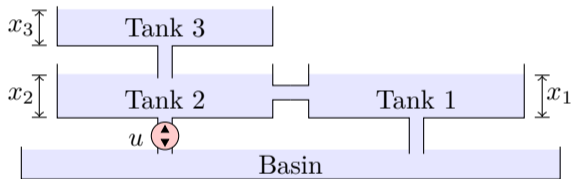


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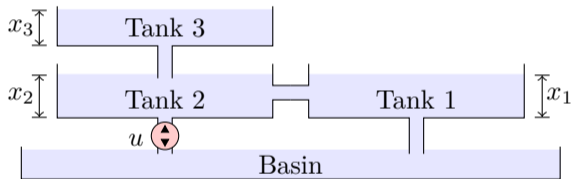


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- ▷ We use the first  $T$  samples for each round of analysis;  $T = 3, 4, 5, 10, 100$ .

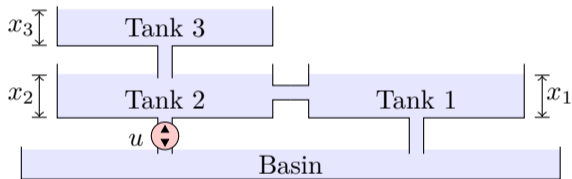


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Table: Informativity of randomly generated data.

$T$	Informative for system identification	$\Sigma_{pk}$ -informative for stabilization	
		$\Sigma_{pk} = \mathcal{M}$	$\Sigma_{pk} = \Sigma_{stab}$
3	0%	8.1%	42%
4	62.4%	63.2%	99.4%
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- ▷ Future work:
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\*A.S., H.J. van Waarde, M.K. Camlibel. “Experiment design using prior knowledge on controllability and stabilizability,” *arXiv preprint*, 2025.



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